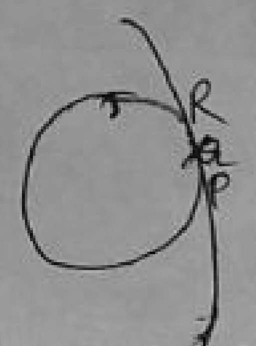
B.Sc Part II (Subsidiary)

# Curvature.

Definition:– Let P be any point on the curve. We take two points Q and R very near to P.

We know that one and only one circle can pass through three points P, Q, R. Now we tend the points Q and R to coincide with P. By so doing there will be a limiting position of the circle PQR. This circle is called the circle of curvature. Hence when Q, R → P, then the limiting position of the circle PQR is called the circle of curvature.




The centre of this circle of curvature is called the centre of curvature and its radius is called the radius of curvature. The chord drawn inside the circle of curvature through P, is called the chord of curvature.

Example– For any curve prove the formula

$$\rho = \frac{r}{\sin\phi\left(1+\frac{d\phi}{d\theta}\right)} \quad \text{Where } \phi = r\frac{d\theta}{dr}.$$

Solution:– We know that

$$\psi = \theta + \phi \quad \therefore \frac{d\psi}{d\theta} = 1 + \frac{d\phi}{d\theta}$$

$$\Rightarrow \frac{d\psi}{ds}\cdot\frac{ds}{d\theta} = 1 + \frac{d\phi}{d\theta} \Rightarrow \frac{1}{\rho}\cdot\frac{ds}{d\theta} = 1 + \frac{d\phi}{d\theta}$$

$$\Rightarrow \frac{1}{\rho}\cdot\frac{ds}{r\,d\theta}\cdot r = 1+\frac{d\phi}{d\theta} \Rightarrow \frac{1}{\rho}\cdot\frac{1}{\sin\phi}\cdot r = 1+\frac{d\phi}{d\theta}$$

$$\rho = \frac{r}{\sin\phi\left(1+\frac{d\phi}{d\theta}\right)}$$

Example– Find the radius of curvature at any point of the curve.

$$x = a(\theta+\sin\theta),\ y = a(1-\cos\theta)$$

Solution:– Given that $x = a(\theta+\sin\theta), y = a(1-\cos\theta)$

$$\frac{dx}{d\theta} = a(1+\cos\theta),\ \frac{dy}{d\theta} = a\sin\theta$$

$$\frac{d^2x}{d\theta^2} = -a\sin\theta,\ \frac{d^2y}{d\theta^2} = a\cos\theta.$$

From the formula.

$$\rho = \frac{(x'^2+y'^2)^{3/2}}{x'y''-y'x''}$$

$$= \frac{\{a^2(1+\cos\theta)^2+a^2\sin^2\theta\}^{3/2}}{a^2\cos\theta(1+\cos\theta)+a^2\sin^2\theta}$$

$$= \frac{a^3(1+2\cos\theta+\cos^2\theta+\sin^2\theta)^{3/2}}{a^2(\cos\theta+\cos^2\theta+\sin^2\theta)}$$

$$= \frac{a(2+2\cos\theta)^{3/2}}{1+\cos\theta} = \frac{a\cdot 2^{3/2}(1+\cos\theta)^{3/2}}{1+\cos\theta}$$

$$= 2^{3/2}\cdot a(1+\cos\theta)^{\frac{1}{2}}$$

$$= 2^{3/2}a\left(2\cos^2\frac{\theta}{2}\right)^{\frac{1}{2}} = 2^{3/2}\cdot 2a\frac{1}{2}\cos\frac{\theta}{2}$$

$$= 4a\cos\frac{\theta}{2}.$$

Ans.

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